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الانسي

د. عرفه

محاضرة 10

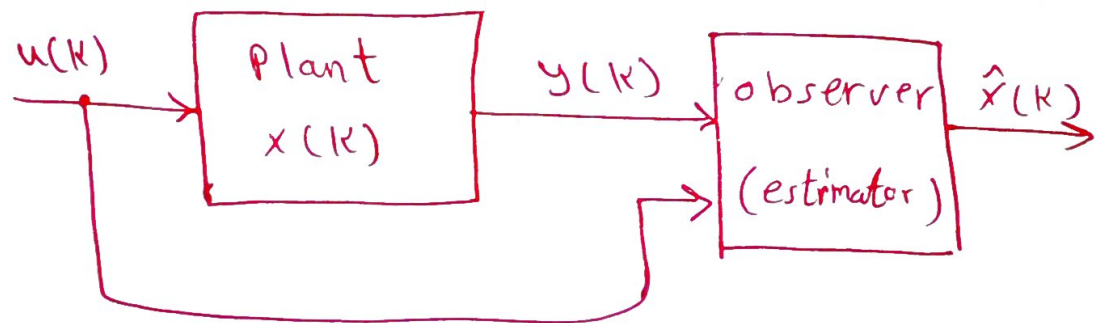
observer Design:-

for state feedback control (Pole Placement design):

$$u(k) = -K x(k)$$

It's required for all states to be accessible for measuring.

In the case of some or all of the states are not accessible for measuring; an observer should be designed to estimate these states values. The estimation is done by the history of data for i/p and o/p



The observer equation:-

$$\hat{x}(k+1) = (A - \underbrace{GK}_{\substack{\text{Gain} \\ \text{Matrix}}}) \hat{x}(k) + Bu(k) + Gy(k)$$

$$\text{gain Matrix } (G) \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}$$

↑ order of the system

The gain matrix G is determined through the specs of the observer

as:
 - observer speed response
 - transient
 - settling time
 } $\xi, \omega_n \Rightarrow$ desired poles of the observer

The Gain matrix G is determined by:

① desired ch. equation for the observer ($\alpha_o(z)$)

$$\alpha_o(z) = (z - p_1)(z - p_2) \dots (z - p_n) = 0 \quad (1)$$

where p_1, p_2, \dots, p_n are the desired poles

$$\hat{x}(k+1) = (A - GC) \hat{x}(k) + Bu(k) + Gy(k)$$

the observer ch. equation

$$|zI - A + GC| = 0 \quad (2)$$

by comparing the parameters of eqn (1) & (2)
 we can get G

② using Ackerman's method:-

$$G = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$\text{for } n=2 \Rightarrow G = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where M_o = observability matrix

$$\alpha_o(A) = \alpha_o(z) \Big|_{z=A}$$

$\alpha_o(z)$ = desired ch. eqn for observer

$$\text{Ex: } x(k+1) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} u(k)$$

$$y(k) = (1 \quad 0) x(k)$$

① determine the gain matrix K such that the desired closed loop poles are located at

$$z_{1,2} = 0.258 \pm j0.295$$

← from previous Lecture (9)

$$K = (0.0775 \quad 0.3945)$$

$$K = (0 \quad 1) M_c^{-1} \alpha_c(A)$$

$$\begin{aligned} \alpha_c(z) &= z^2 - 1.0562z + 0.366 \\ \alpha_c(A) &= \begin{pmatrix} 0.31 & 1.888 \\ 0 & 0.31 \end{pmatrix} \\ M_c^{-1} &= \begin{pmatrix} -0.25 & 0.75 \\ 0.25 & -0.25 \end{pmatrix} \end{aligned}$$

② design a full order observer such that $\xi = 1$ & the time constant of the observer

is one half the time constant of the desired poles in ①

$$T = 1 \text{ sec.}$$

$$\boxed{z_0 = 1/2 z_c}$$

①?

$$s_{1,2} = \xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}$$

$$\Rightarrow z_{1,2} = r \angle \pm \theta = r \cos \theta \pm j r \sin \theta$$

$$z_{1,2} = 0.528 \pm j0.295$$

$$\begin{aligned} \Downarrow \\ r &= \sqrt{0.528^2 + 0.295^2} \\ &= 0.605 \end{aligned}$$

$$\begin{aligned} r &= e^{-\xi \omega_n T} = e^{-T/\tau_c} \\ \theta &= \pm \omega_n T \sqrt{1 - \xi^2} \end{aligned}$$

$$r = e^{-\xi \omega_n T} = e^{-T/\tau_c}$$

$$\ln r = -\frac{T}{\tau_c}$$

$$\ln(0.605) = -\frac{1}{\tau_c} \Rightarrow \tau_c \approx \text{sec}$$

$$\tau_o = \frac{1}{2} (2) = 1 \text{ sec.}$$

* the desired poles of the observer.

$$\tau_o = 1 \text{ sec.} \& \zeta = 1$$

$$\tau_o = \frac{1}{\xi \omega_n}$$

$$1 = \frac{1}{\omega_n} \Rightarrow \omega_n = 1$$

$$r = e^{-\xi \omega_n T} = e^{-\frac{T}{\tau_o}}$$

$$= e^{-1} = 0.3678$$

$$\theta = \omega_n T \sqrt{1 - \xi^2} = 0$$

$$z_{1,2} = r (\cos \theta \pm j \sin \theta)$$

$$= r = 0.3678$$

the desired ch. eq for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.3678)^2$$

$$\alpha_o(z) = z^2 - 0.7356z + 0.1353$$

$$\alpha_o(A) = A^2 - 0.7356A + 0.1353I$$

$$= \begin{pmatrix} 0.3997 & 2.5288 \\ 0 & 0.3997 \end{pmatrix}$$

$$G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} : M_o(cA)$$

$$M_o = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \Rightarrow M_o^{-1} = \begin{pmatrix} 1 & 0 \\ -0.5 & 0.5 \end{pmatrix}$$

$$G = \begin{pmatrix} 1.2644 \\ 0.1998 \end{pmatrix}$$

$$\text{Ex 2 : } x(k+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = (1 \quad 0) x(k)$$

design a full order observer such that :

- the desired observer poles must be real
- the time constant is 10 sec
- $T = 1$ sec

$\xi = 1$ \rightarrow overdamped

* desired poles of the observer:-

$$r = e^{-T/\tau} = e^{-1/10} = e^{-0.1} = 0.905$$

$$\theta = \omega_n T \sqrt{1 - \xi^2} = 0 \quad (\text{Real and equal})$$

$$z_{1,2} = r = 0.905 \quad \xi = 1$$

* the desired ch. eqn for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.905)^2 = z^2 - 1.81z + 0.819$$

$$\alpha_o(A) = \begin{pmatrix} 0.009 & 0.19 \\ 0 & 0.009 \end{pmatrix}$$

$$G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.19 \\ 0.009 \end{pmatrix}$$

$$M_o = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$M_o^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$